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M/CS 335

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**M/CS 335 Project 1**

Hello Molly, I have analyzed the data that you gave to me about your biscuit manufacturing business and have come up with two models that meet your two different sets of requirements. I believe that if you follow these two models then you will be able to achieve the highest profit margins while also bringing satisfaction to your customers around the world.

Having monthly shipping requirements makes sense when it comes to customer satisfaction, but I believe I can show that even taking the losses associated with not shipping enough product each month, more money can be made selling the same amount annually. But first let me run through the model requiring the demands to be met.

For the variables I have chosen Mi for i = 1,2,3,...,12 to be the amount of biscuits manufactured in month i. The constraints meet the requirements for nonnegativity, production limits, and monthly demand. Amount remaining after each month, Hi for i= 1,2,3,...,12 are calculated by taking the amount in stock after demands are met each month i. Our objective function multiplies the cost of production in each month by the amount made, and then takes the sum. The constraints that allows us to keep track of costs follow a simple rule outlined in our book. For each month i, (the amount in stock + the amount made) = (the amount sold + the amount left over). A formal model is as follows:

Minimum cost = 6x(M1 + M2 + M5 + M12) + 7x(M3 + M4) + 5x(M6 + M11) + 4x(M7 + M8 + M9) + 3x(M10) + 2x(H1) + 3x(H2) + 4x(H3) + 3x(H4) + 3x(H5) + 3x(H6) + 2x(H7) + 2x(H8) + 2x(H9) + 3x(H10) + 3x(H11) + 4x(H12),

s.t.

M1 = 50 + H1,

H1 + M2 = 45 + H2,

H2 + M3 = 40 + H3,

H3 + M4 = 30 + H4,

H4 + M5 = 20 + H5,

H5 + M6 = 50 + H6,

H6 + M7 = 70 + H7,

H7 + M8 = 80 + H8,

H8 + M9 = 60 + H9,

H9 + M10 = 45 + H10,

H10 + M11 = 30 + H11,

H11 + M12 = 40 + H12,

Hi >= 0 for all i

50 <= m1 <= 70, 0 <= m2 <= 50, 0 <= m3 <= 30, 0 <= m4 <= 40, 0 <= m5 <= 50, 0 <= m6 <= 40, 0 <= m7 <= 50,

0 <= m8 <= 60, 0 <= m9 <= 80, 0 <= m10 <= 60, 0 <= m11 <= 50, 0 <= m12 <= 20},

Mi = amount of biscuits made in month i

Hi = biscuits to be held after month i

To analyze this model I used Mathematica solve it, and then I put the optimal manufacturing/holding schedule in a table.

|  |  |  |  |
| --- | --- | --- | --- |
| Month: | Amount to produce  (cases): | Amount to hold for next month (cases): | Optimal cost  (dollars): |
| 1 | 65 | 15 | 3425 |
| 2 | 50 | 20 |  |
| 3 | 30 | 10 |  |
| 4 | 40 | 20 |  |
| 5 | 50 | 50 |  |
| 6 | 40 | 40 |  |
| 7 | 50 | 20 |  |
| 8 | 60 | 0 |  |
| 9 | 60 | 0 |  |
| 10 | 45 | 0 |  |
| 11 | 50 | 20 |  |
| 12 | 20 | 0 |  |

This schedule will guarantee you the lowest cost of production over the course of one year with the requirements that you gave me for part one. As you can see, towards the second half of the year as demands increase it isn’t possible to hold as much product in stock as it would be earlier in the year. This is okay because your production capacity increases to meet the demands. This just means your inventory is moving out the door and your holding costs are down. In the early months it is more cost effective to hold more when production values are cheap. This plan is effective and also will make all of your customers happy, however looking at the second set of requirements I believe we can come up with a plan to make more money.

Your second set of requirements proved to be slightly more difficult but could be solved using the same basic outline as our previous model plus another couple sets of variables to keep track of things. The new variables Di for i=1,2,3,...,12 are the deficit after month i, and Ti, i=1,2,3,...,12 is the amount sold in month i since we can now choose how much of the demand we want to meet. The objective function is the same but adding on the costs for deficits onto it. The new constraints follow the same basic principles as before but now instead of having the demand built in, we replace it with a variable for amount sold. That combined with letting the deficit equal the demand - amount sold lets our model decide how much to sell each month. The formal model is as follows:

Minimum cost = 6x(M1 + M2 + M5 + M12) + 7x(M3 + M4) + 5x(M6 + M11) + 4x(M7 + M8 + M9) + 3x(M10) + 2x(H1) + 3x(H2) + 4x(H3) + 3x(H4) + 3x(H5) + 3x(H6) + 2x(H7) + 2x(H8) + 2x(H9) + 3x(H10) + 3x(H11) + 4x(H12) + 9x(D1+D2+D3+D4+D5+D6) + 12X(D7 + D8 + D9 + D10 + D11 + D12),

s.t.

H1 = M1 - T1,

H2 = (H1 + M2) - T2,

H3 = (H2 + M3) - T3,

H4 = (H3 + M4) - T4,

H5 = (H4 + M5) - T5,

H6 = (H5 + M6) - T6,

H7 = (H6 + M7) - T7,

H8 = (H7 + M8) - T8,

H9 = (H8 + M9) - T9,

H10 = (H9 + M10) - T10,

H11 = (H10 + M11) - T11,

H12 = (H11 + M12) - T12,

D1 = 50 - T1,

D2 = 45 - T2,

D3 = 40 - T3,

D4 = 30 - T4,

D5 = 20 - T5,

D6 = 50 - T6,

D7 = 70 - T7,

D8 = 80 - T8,

D9 = 60 - T9,

D10 = 45 - T10,

D11 = 30 - T11,

D12 = 40 - T12,

Hi >= 0 for all i,

Di >= 0 for all i,

Ti >= 0 for all i

50 <= m1 <= 70, 0 <= m2 <= 50, 0 <= m3 <= 30, 0 <= m4 <= 40, 0 <= m5 <= 50, 0 <= m6 <= 40, 0 <= m7 <= 50,

0 <= m8 <= 60, 0 <= m9 <= 80, 0 <= m10 <= 60, 0 <= m11 <= 50, 0 <= m12 <= 20},

Mi = amount of biscuits made in month i,

Hi = biscuits to be held after month i,

Di = deficit after month i,

Ti = amount sold in month i

To analyze this model I used Mathematica solve it, and then I put the optimal manufacturing, holding, and selling schedule in a table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Month: | Amount to produce (cases): | Amount to sell  (cases): | Amount to hold  (cases): | Optimal cost  (dollars): |
| 1 | 50 | 50 | 0 | 3215 |
| 2 | 45 | 45 | 0 |  |
| 3 | 30 | 30 | 0 |  |
| 4 | 30 | 30 | 0 |  |
| 5 | 20 | 20 | 0 |  |
| 6 | 40 | 20 | 20 |  |
| 7 | 50 | 70 | 0 |  |
| 8 | 60 | 60 | 0 |  |
| 9 | 60 | 60 | 0 |  |
| 10 | 45 | 45 | 0 |  |
| 11 | 50 | 30 | 0 |  |
| 12 | 20 | 40 | 0 |  |

Not meeting the demands in months 3, 6, and 8 is actually cheaper than over producing in previous months and paying production and holding costs.This boils down to it being cheaper to not meet demands in months where it would be costly to do so. This “costly” factor is the sum of the previous manufacturing costs as well as the holding costs but our model balanced those with the cost of a deficit. The downside to this is simply that the demand is not met. In the long run this could have negative effects on your business and I would advise against it, however it does add up to less costs and more profit.

In conclusion, I would say that the two models meeting your requirements should give you a pretty decent idea of how to move forward with your business plan. It is complicated to balance the holding and production costs and this makes it difficult to know when to sell but I believe that following either of these schedules that I have given you will minimize costs and maximize profits while bringing delicious dog biscuits to the world.

**Appendix:**

**Problem 1 input:**

Minimize[{6\*(m1 + m2 + m5 + m12) + 7\*(m3 + m4) + 5\*(m6 + m11) +

4\*(m7 + m8 + m9) + 3\*(m10) + 2\*(h1) + 3\*(h2) + 4\*(h3) + 3\*(h4) +

3\*(h5) + 3\*(h6) + 2\*(h7) + 2\*(h8) + 2\*(h9) + 3\*(h10) + 3\*(h11) +

4\*(h12),

0 + m1 == 50 + h1,

h1 + m2 == 45 + h2,

h2 + m3 == 40 + h3,

h3 + m4 == 30 + h4,

h4 + m5 == 20 + h5,

h5 + m6 == 50 + h6,

h6 + m7 == 70 + h7,

h7 + m8 == 80 + h8,

h8 + m9 == 60 + h9,

h9 + m10 == 45 + h10,

h10 + m11 == 30 + h11,

h11 + m12 == 40 + h12,

0 <= h1,

0 <= h2,

0 <= h3,

0 <= h4,

0 <= h5,

0 <= h6,

0 <= h7,

0 <= h8,

0 <= h9,

0 <= h10,

0 <= h11,

0 <= h12,

50 <= m1 <= 70,

0 <= m2 <= 50,

0 <= m3 <= 30,

0 <= m4 <= 40,

0 <= m5 <= 50,

0 <= m6 <= 40,

0 <= m7 <= 50,

0 <= m8 <= 60,

0 <= m9 <= 80,

0 <= m10 <= 60,

0 <= m11 <= 50,

0 <= m12 <= 20},

{m1, m2, m3, m4, m5, m6, m7, m8, m9, m10, m11, m12, h1, h2, h3, h4,

h5, h6, h7, h8, h9, h10, h11, h12}]

**Problem 1 output:**

{3425, {m1 -> 65, m2 -> 50, m3 -> 30, m4 -> 40, m5 -> 50, m6 -> 40,

m7 -> 50, m8 -> 60, m9 -> 60, m10 -> 45, m11 -> 50, m12 -> 20,

h1 -> 15, h2 -> 20, h3 -> 10, h4 -> 20, h5 -> 50, h6 -> 40,

h7 -> 20, h8 -> 0, h9 -> 0, h10 -> 0, h11 -> 20, h12 -> 0}}

**Problem 2 input:**

Minimize[{6\*(m1 + m2 + m5 + m12) + 7\*(m3 + m4) + 5\*(m6 + m11) +

4\*(m7 + m8 + m9) + 3\*(m10) + 2\*(h1) + 3\*(h2) + 4\*(h3) + 3\*(h4) +

3\*(h5) + 3\*(h6) + 2\*(h7) + 2\*(h8) + 2\*(h9) + 3\*(h10) + 3\*(h11) +

4\*(h12) + 9\*((d1) + (d2) + (d3) + (d4) + (d5) + (d6)) +

12\*((d7) + (d8) + (d9) + (d10) + (d11) + (d12)),

h1 == m1 - t1,

h2 == (h1 + m2) - t2,

h3 == (h2 + m3) - t3,

h4 == (h3 + m4) - t4,

h5 == (h4 + m5) - t5,

h6 == (h5 + m6) - t6,

h7 == (h6 + m7) - t7,

h8 == (h7 + m8) - t8,

h9 == (h8 + m9) - t9,

h10 == (h9 + m10) - t10,

h11 == (h10 + m11) - t11,

h12 == (h11 + m12) - t12,

d1 == 50 - t1,

d2 == 45 - t2,

d3 == 40 - t3,

d4 == 30 - t4,

d5 == 20 - t5,

d6 == 50 - t6,

d7 == 70 - t7,

d8 == 80 - t8,

d9 == 60 - t9,

d10 == 45 - t10,

d11 == 30 - t11,

d12 == 40 - t12,

0 <= h1,

0 <= h2,

0 <= h3,

0 <= h4,

0 <= h5,

0 <= h6,

0 <= h7,

0 <= h8,

0 <= h9,

0 <= h10,

0 <= h11,

0 <= h12,

0 <= d1,

0 <= d2,

0 <= d3,

0 <= d4,

0 <= d5,

0 <= d6,

0 <= d7,

0 <= d8,

0 <= d9,

0 <= d10,

0 <= d11,

0 <= d12,

0 <= t1 <= 50,

0 <= t2 <= 45,

0 <= t3 <= 40,

0 <= t4 <= 30,

0 <= t5 <= 20,

0 <= t6 <= 50,

0 <= t7 <= 70,

0 <= t8 <= 80,

0 <= t9 <= 60,

0 <= t10 <= 45,

0 <= t11 <= 30,

0 <= t12 <= 40,

0 <= m1 <= 70,

0 <= m2 <= 50,

0 <= m3 <= 30,

0 <= m4 <= 40,

0 <= m5 <= 50,

0 <= m6 <= 40,

0 <= m7 <= 50,

0 <= m8 <= 60,

0 <= m9 <= 80,

0 <= m10 <= 60,

0 <= m11 <= 50,

0 <= m12 <= 20},

{m1, m2, m3, m4, m5, m6, m7, m8, m9, m10, m11, m12, h1, h2, h3, h4,

h5, h6, h7, h8, h9, h10, h11, h12, d1, d2, d3, d4, d5, d6, d7, d8,

d9, d10, d11, d12, t1, t2, t3, t4, t5 , t6, t7, t8, t9, t10, t11,

t12}]

**Problem 2 output:**

{3215, {m1 -> 50, m2 -> 45, m3 -> 30, m4 -> 30, m5 -> 20, m6 -> 40,

m7 -> 50, m8 -> 60, m9 -> 60, m10 -> 45, m11 -> 50, m12 -> 20,

h1 -> 0, h2 -> 0, h3 -> 0, h4 -> 0, h5 -> 0, h6 -> 20, h7 -> 0,

h8 -> 0, h9 -> 0, h10 -> 0, h11 -> 20, h12 -> 0, d1 -> 0, d2 -> 0,

d3 -> 10, d4 -> 0, d5 -> 0, d6 -> 30, d7 -> 0, d8 -> 20, d9 -> 0,

d10 -> 0, d11 -> 0, d12 -> 0, t1 -> 50, t2 -> 45, t3 -> 30,

t4 -> 30, t5 -> 20, t6 -> 20, t7 -> 70, t8 -> 60, t9 -> 60,

t10 -> 45, t11 -> 30, t12 -> 40}}